EPSRC OxPDE CDT Research Case Study Nikolaos Athanasiou (Cohort 3 2016-2020)

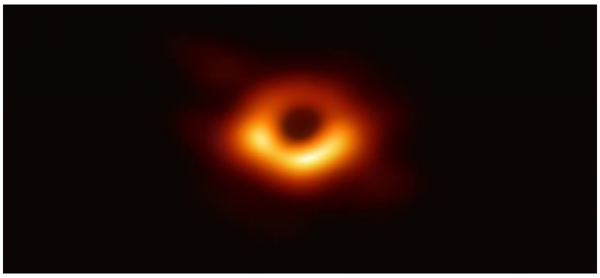
“Studies on the Formation of Black Holes and Singularities in Relativity”

Few notions within mathematical physics inspire awe and capture the imagination as much as that of a black hole. The origins of our study of black holes can be traced back to a letter from Karl Schwarzschild to Albert Einstein in 1915. Within that letter was the first, non-trivial solution to the Einstein equations of General Relativity, the celebrated Schwarzschild solution:

$$g = -(1-\frac{2M}{r})dt^2 + (1-\frac{2M}{r})^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2).$$

It took 30 years before the contents of this letter were properly understood, which y contained the first theoretical example of a black hole.

What is, in essence, a black hole? Think of a region of space where gravity is so strong that nothing (not even light) can escape. Until recently, the only pictures of black holes could be found in drawings and their existence was disputed. This radically changed after the Event Horizon Telescope succeeded in capturing the first black hole image:



Event Horizon Telescope, April 2019

We now know that black holes do exist - *but are the actual properties of real black holes the same as the properties our mathematical models predict?* To answer this mathematicians have **3 "tests of reality" for black holes: Stability, formation & rigidity.** My research revolves around the first two.

1) **Black hole stability:** Imagine throwing a pebble into a black hole. Do you expect to destroy it or change nothing? For black holes to be real, the latter must be true. Mathematically, this is difficult to address, and we do so by perturbing the initial data around a fixed state we know leads to a well-known black hole, and hope that upon evolution the perturbed data give rise to a space-time with much the same properties as the black hole.

2) **Black hole formation:** For black holes to be physical, they should form dynamically, i.e. from a time when black holes did not exist. This is the content of the "formation" test of reality. Mathematically, we again adopt the initial data approach and hope to prove the formation of certain objects called *trapped surfaces* (for which Sir Roger Penrose, Oxford won the 2020 Nobel Prize for Physics). This is far from easy & requires deep analysis of Einstein’s equations. A third aspect of my work revolves around the understanding of fluid dynamics under a General (or Special) relativistic viewpoint.

My research attempts to reconcile physical reality with mathematical reality in the special case of black holes. This is vital, as mathematical models could then infer many properties about black holes and guide experimental scientists with their research. I am grateful to the EPSRC for enabling me to add to the body of knowledge concerning black holes.